Title: Relationships Among Arc and Angle Measures in Circles

Grade Ranges:
___K-4
___5-8
_X_9-12

Subject Tag:
Math: Geometry

Synopsis:
This activity can be used as an exploration of new material. It can also be used after discussion of central angles and inscribed angles and can be students' introduction to the relationships that exist among other angle and arc measures in circles. Using the computer program Geometer's Sketchpad, students will complete five different constructions of circles provided in the accompanying handout. Each construction includes radii, tangents, secants, and/or chords. A series of steps guides the students through each construction, indicating which angles and arcs to measure. One or two questions follow each construction by asking students to generalize and deduce from their sketch. Students will discover relationships among various angle and arc measures in circles. They will also explore semicircles and angles inscribed in quadrilaterals. Note: this lesson plan can be modified for other programs such as The Geometric Supposer and Pre-Supposer series or for Cabri Geometry.

Keywords:
circles, angles, arcs, tangents, secants, chords, semicircles, inscribed, radius, intercepted, Geometer's Sketchpad

Body:

All constructions are described in the accompanying handout.

1. To get students started on the right track, the first construction should be done together. Students should be at their computers, and ideally you should project your construction on a screen. Students should perform the construction themselves in addition to watching you construct yours.

2. For the four remaining constructions, students usually work best when allowed to work in pairs or give one another help. As students work, circulate around the room, giving help when necessary.

3. When everyone has finished the constructions and answered the questions on the handout, discuss the questions. It should not be necessary to go through all of the constructions. Rather, focus on the conclusions that the students are able to draw from the constructions and the questions. Encourage students to correct and add to their answers during discussion if necessary.

Related Links:

Math Forum: Ask Dr. Math
http://mathforum.org/dr.math/problems/sondra.11.07.01.html
This particular page shows students how to prove that an angle inscribed in a semicircle is a right angle.

Math Forum: Teaching and learning of the circle theorems
http://www.mathforum.org/t2t/thread.taco?thread=6278
This is another link to a part of the "Math Forum" site, but this one is in a category called "Teacher2Teacher." This particular dialogue is about how to teach the circle theorems and possible activities to use in introducing them.

Key Curriculum Press: The Geometer's Sketchpad
http://www.keypress.com/catalog/products/software/Prod_GSP.html
This official product site contains information on the capabilities of Sketchpad and other computer activities.

Features:
__Contains special education tips
___Quick Activity (less than 30 minutes; story starter)
___Requires Internet access for students to complete

Objectives:
Using Geometer's Sketchpad, students will complete five different constructions of circles, and in the process discover relationships among various angle and arc measures in circles.

Standards:
NY: 1. Mathematical Reasoning. Students use mathematical reasoning to analyze mathematical situations, make conjectures, gather evidence, and construct an argument. **7.** Patterns/Functions. Students use patterns and functions to develop mathematical power, appreciate the true beauty of mathematics, and construct generalizations that describe patterns simply and efficiently.

NYC: M2b. Works with two- and three-dimensional figures and their properties, including polygons and circles, cubes and pyramids, and cylinders, cones, and spheres. **M2q.** Explores geometry using computer programs such as CAD software, Sketchpad programs, or LOGO. **M5c.** Conclusion: The student provides closure to the solution process through summary statements and general conclusions. **M6d.** Uses basic geometric terminology accurately, and deduces information about basic geometric figures in solving problems. **M7e.** Presents mathematical ideas effectively both orally and in writing.

CT: 6. Spatial Relationships and Geometry. Students will analyze and use spatial relationships and basic concepts of geometry to construct, draw, describe and compare geometric models and their transformations, and use geometric relationships and patterns to solve problems.

NJ: 4.4. All students will develop reasoning ability and will become self-reliant, independent mathematical thinkers. **4.5.** All students will regularly and routinely use calculators, computers, manipulatives, and other mathematical tools to enhance mathematical thinking, understanding, and power.

Prerequisite Skills:

1. Familiarity with basic Spetchpad use
2. Knowledge of circles and related concepts (tangent, chord, radius, arc, etc.)

Time Required:

50 minutes

Technology and Materials Needed:

1. Geometer's Sketchpad, with one or two students per computer

Assessment Criteria:

1. Collect the handout after discussion of the questions. Grades should be based upon completion. Grades should also reflect the amount of detail included in the corrections of / additions to answers during class discussion.

Recommended Lesson Plan Review Date:

Review Comments:

Check Web sites.

Relationships Among Arc and Angle Measures in Circles

General Directions: Do the constructions described below on Geometer's Sketchpad. Remember that "construct" means to use the "Construct" menu. Record all necessary arc and angle measures on this sheet. After completing each construction, answer the question(s) that follow it before moving on to the next construction. Do not delete any sketches until you are instructed to start a new sketch.

Construction 1:

1. Construct circle *A*.
2. Construct radius $\overline{AB}$, and make this segment dashed by using the "Display" menu.
3. Construct a line tangent to the circle at point *B*. (Hint: this tangent will be perpendicular to $\overline{AB}$.)
4. Construct a point *C* on the circle.
5. If necessary, drag point *C* so it is fewer than 180 degrees counterclockwise from point *B*.
6. Construct chord $\overline{BC}$.
7. Construct minor arc *BC* by selecting point *B*, point *C*, and the circle (in that order) and then by going to the "Construct" menu.
8. While the arc (that you just constructed) is selected, use the "Display" menu to make it thick.
9. Measure arc *BC* by selecting "Arc angle" on the "Measure" menu.

   m arc *BC* = _____

10. Construct point *D* on the tangent line.
11. If necessary, drag *D* so that $\angle CBD$ (the angle formed by the chord and the tangent) is acute.
12. Measure the acute angle, $\angle CBD$, again using the "Measure" menu.

    $m\angle CBD$ = _____

13. Repeat steps 7 - 12, but this time use the *major* arc and the *obtuse* angle. (Hint: to construct the major arc, select point *C* first; to create the obtuse angle, create a point *E* on the tangent line.)

    m major arc = _____

    $m\angle CBE$ = _____

    What is the sum of the two arc measures? _____

    What is the sum of the two angle measures? _____

Question: How would you describe the relationship between the measure of an angle formed by a tangent and a chord (which intersect on a circle) and the measure of that angle's intercepted arc?

Construction 2:

1. Begin a new sketch.
2. Construct circle *A*.
3. Construct two intersecting chords.
4. Construct their point of intersection.

5. Measure any one of the four angles formed by the chords.

   Angle measure = _____

6. Measure that angle's intercepted arc and its vertical angle's intercepted arc.

   Measure of intercepted arc = _____

   Measure of vertical angle's intercepted arc = _____

7. Repeat steps 5 and 6 for all four angles formed by the chords.

Question: How would you describe the relationship between the measure of an angle formed by intersecting chords and the measures of the arcs that it and its vertical angle intercept?

Construction 3:

1. Begin a new sketch.
2. Construct circle *A*.
3. Construct point *C* on the circle.
4. Construct two radii, $\overline{AB}$ and $\overline{AC}$.
5. Construct two tangent lines to the circle, one with point of tangency at *B* and the other with point of tangency at *C*.
6. Construct the point of intersection of the tangent lines. If you cannot see their point of intersection, drag either *B* or *C* around the circle until it appears.
7. Measure the angle formed by the tangent lines. (You want to measure the angle whose vertex is their point of intersection and whose sides go through *B* and *C*.)

   Angle measure = _____

8. Construct minor arc *BC*, and also construct the major arc formed by points *B* and *C*, as you did previously in Construction 1.
9. Measure these two arcs created by the points of tangency.

   m arc *BC* = _____

   Measure of major arc = _____

   What is the sum of the measures of these two arcs? _____

Question: How would you describe the relationship between the measure of the angle formed by two tangents and the measures of the arcs that those tangents intercept?

Construction 4:

1. Begin a new sketch.
2. Construct circle *A*.
3. Construct two secants that intersect in the *exterior* of the circle.
4. Construct their point of intersection.
5. Measure the angle formed by the two secants.

   Angle measure = _____

6. Measure the two intercepted arcs.

   Measure of first intercepted arc = _____

   Measure of second intercepted arc = _____

Question A: How would you describe the relationship between the measure of the angle formed by two secants and the measures of the intercepted arcs?

Question B: What do you suppose would happen if you repeated steps 1 - 6 using a secant and a tangent?

Construction 5:

1. Begin a new sketch.
2. Construct circle *A*.
3. Construct an inscribed quadrilateral BCDE.
4. Measure all four angles of the quadrilateral.

   $m\angle CBE$ _____

   $m\angle BCD$ = _____

   $m\angle CDE$ = _____

   $m\angle BED$ = _____

5. Drag one or more of the vertices of the quadrilateral, and observe the measures of its angles.

Question: How would you describe the relationship among the measures of the angles of an inscribed quadrilateral?